

## NAG C Library Function Document

### nag\_prob\_non\_central\_f\_dist (g01gdc)

#### 1 Purpose

nag\_prob\_non\_central\_f\_dist (g01gdc) returns the probability associated with the lower tail of the non-central  $F$  or variance-ratio distribution.

#### 2 Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_prob_non_central_f_dist(double f, double df1, double df2,
    double lambda, double tol, Integer max_iter, NagError *fail)
```

#### 3 Description

The lower tail probability of the non-central  $F$ -distribution with  $\nu_1$  and  $\nu_2$  degrees of freedom and non-centrality parameter  $\lambda$ ,  $P(F \leq f : \nu_1, \nu_2; \lambda)$ , is defined by

$$P(F \leq f : \nu_1, \nu_2; \lambda) = \int_0^x p(F : \nu_1, \nu_2; \lambda) dF$$

where

$$P(F : \nu_1, \nu_2; \lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)^j}{j!} \times \frac{(\nu_1 + 2j)^{(\nu_1+2j)/2} \nu_2^{\nu_2/2}}{B((\nu_1 + 2j)/2, \nu_2/2)} \\ \times u^{(\nu_1+2j-2)/2} [\nu_2 + (\nu_1 + 2j)u]^{-(\nu_1+2j+\nu_2)/2}$$

and  $B(\cdot, \cdot)$  is the beta function.

The probability is computed by means of a transformation to a non-central beta distribution;

$$P(F \leq f : \nu_1, \nu_2; \lambda) = P_{\beta}(X \leq x : a, b; \lambda)$$

where  $x = \frac{\nu_1 f}{\nu_1 f + \nu_2}$  and  $P_{\beta}(X \leq x : a, b; \lambda)$  is the lower tail probability integral of the non-central beta distribution with parameters  $a$ ,  $b$ , and  $\lambda$ .

If  $\nu_2$  is very large, greater than  $10^6$ , then a  $\chi^2$  approximation is used.

#### 4 Parameters

- |    |  |              |
|----|--|--------------|
| 1: | <b>f</b> – double  | <i>Input</i> |
|    | <i>On entry:</i> the deviate from the non-central $F$ -distribution, $f$ .     |              |
|    | <i>Constraint:</i> <b>f</b> > 0.   |              |
| 2: | <b>df1</b> – double  | <i>Input</i> |
|    | <i>On entry:</i> the degrees of freedom of the numerator variance, $\nu_1$ .   |              |
|    | <i>Constraint:</i> $0.0 < \mathbf{df1} \leq 1.0e6$ .                           |              |
| 3: | <b>df2</b> – double  | <i>Input</i> |
|    | <i>On entry:</i> the degrees of freedom of the denominator variance, $\nu_2$ . |              |
|    | <i>Constraint:</i> <b>df2</b> > 0.0.   |              |

- 4: **lambda** – double *Input*  
*On entry:* the non-centrality parameter,  $\lambda$ .  
*Constraint:*  $0.0 \leq \mathbf{lambda} \leq -2.0 \times \log(U)$  where  $U$  is the safe range parameter as defined by `nag_real_safe_small_number` (X02AMC).
- 5: **tol** – double *Input*  
*On entry:* the relative accuracy required by the user in the results. If `nag_prob_non_central_f_dist` is entered with **tol** greater than or equal to 1.0 or less than  $10 \times \mathbf{machine\ precision}$  (see `nag_machine_precision` (X02AJC)), then the value of  $10 \times \mathbf{machine\ precision}$  is used instead.
- 6: **max\_iter** – Integer *Input*  
*On entry:* the maximum number of iterations to be used.  
*Suggested value:* 500. See `nag_prob_non_central_chi_sq` (g01gcc) and `nag_prob_non_central_beta_dist` (g01gec) for further details.  
*Constraint:*  $\mathbf{max\_iter} \geq 1$ .
- 7: **fail** – NagError \* *Input/Output*  
The NAG error parameter (see the Essential Introduction).

## 5 Error Indicators and Warnings

### NE\_REAL\_ARG\_CONS

On entry, **df1** = *<value>*.

This parameter must satisfy  $0.0 < \mathbf{df1} \leq 1.0e6$ .

On entry, **lambda** = *<value>*.

This parameter must satisfy  $0.0 \leq \mathbf{lambda} \leq -2.0 * \log(\mathbf{X02AMC})$ .

### NE\_REAL\_ARG\_LE

On entry, **df2** must not be less than or equal to 0.0: **df2** = *<value>*.

On entry, **f** must not be less than or equal to 0.0: **f** = *<value>*.

### NE\_INT\_ARG\_LT

On entry, **max\_iter** must not be less than 1: **max\_iter** = *<value>*.

### NE\_CONV

The solution has failed to converge in *<value>* iterations, consider increasing **max\_iter** or **tol**.

### NE\_PROB\_F

The required probability cannot be computed accurately. This may happen if the result would be very close to zero or one. Alternatively the values of **df1** and **f** may be too large. In the latter case the user could try using a normal approximation, see Abramowitz and Stegun (1972).

### NE\_PROB\_F\_INIT

The required accuracy was not achieved when calculating the initial value of the central  $F$  or  $\chi^2$  probability. The user should try a larger value of **tol**. If the  $\chi^2$  approximation is being used then `nag_prob_non_central_f_dist` returns zero otherwise the value returned should be an approximation to the correct value.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

**6 Further Comments**

When both  $\nu_1$  and  $\nu_2$  are large a normal approximation may be used and when only  $\nu_1$  is large a  $\chi^2$  approximation may be used. In both cases  $\lambda$  is required to be of the same order as  $\nu_1$ . See Abramowitz and Stegun Abramowitz and Stegun (1972) for further details.

**6.1 Accuracy**

The relative accuracy should be as specified by **tol**. For further details see `nag_prob_non_central_chi_sq` (g01gcc) and `nag_prob_non_central_beta_dist` (g01gec).

**6.2 References**

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Dover Publications (3rd Edition)

**7 See Also**

`nag_prob_non_central_chi_sq` (g01gcc)  
`nag_prob_non_central_beta_dist` (g01gec)

**8 Example**

Values from, and degrees of freedom for  $F$ -distributions are read, the lower-tail probabilities computed, and all these values printed, until the end of data is reached.

**8.1 Program Text**

```
/* nag_prob_non_central_f_dist (g01gdc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */

#include <stdio.h>
#include <nag.h>
#include <nagg01.h>

int main(void)
{
    double df1, df2, f, prob, lambda, tol;
    Integer max_iter;
    Integer exit_status=0;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("g01gdc Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[\n]");
```

```

Vprintf ("\n      f      df1      df2      lambda      prob\n\n");
tol = 5e-6;
max_iter = 50;
while ((scanf("%lf %lf %lf %lf %*[\n]", &f, &df1, &df2, &lambda)) != EOF)
{
    prob = g01gdc(f, df1, df2, lambda, tol, max_iter, &fail);
    if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g01gdc.\n%s\n", fail.message);
    exit_status=1;
    goto END;
}
    Vprintf("%8.3f %8.3f %8.3f %8.3f %8.4f\n", f, df1, df2, lambda, prob);
}
END:
return exit_status;
}

```

## 8.2 Program Data

g01gdc Example Program Data

5.5	1.5	25.5	3.0	:f df1 lambda
39.9	1.0	1.0	2.0	:f df1 lambda
2.5	20.25	1.0	0.0	:f df1 lambda

## 8.3 Program Results

g01gdc Example Program Results

f	df1	df2	lambda	prob
5.500	1.500	25.500	3.000	0.8214
39.900	1.000	1.000	2.000	0.8160
2.500	20.250	1.000	0.000	0.5342

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